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Mexican high school students' social representations of mathematics, its teaching and learning

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This paper reports a qualitative research that identifies Mexican high school students' social representations of mathematics. For this purpose, the social representations of 'mathematics', 'learning mathematics' and 'teaching mathematics' were identified in a group of 50 students. Focus group interviews were carried out in order to obtain the data. The constant comparative style was the strategy used for the data analysis because it allowed the categories to emerge from the data. The students' social representations are: (A) Mathematics is. . .(1) important for daily life, (2) important for careers and for life, (3) important because it is in everything that surrounds us, (4) a way to solve problems of daily life, (5) calculations and operations with numbers, (6) complex and difficult, (7) exact and (6) a subject that develops thinking skills; (B) To learn mathematics is. . .(1) to possess knowledge to solve problems, (2) to be able to solve everyday problems, (3) to be able to make calculations and operations, and (4) to think logically to be able to solve problems; and (C) To teach mathematics is. . .(1) to transmit knowledge, (2) to know to share it, (3) to transmit the reasoning ability, and (4) to show how to solve problems.

Keywords: views of mathematics; high school students; social representation; metaphorical thinking; beliefs; conceptions

1. Introduction

There has been great interest in the views of students and teachers about mathematics, its teaching and learning in the field of mathematics education for at least three decades. In an international context, the teachers' and students' views of mathematics have been identified through concepts such as: *conceptions* [1–9], *beliefs* [10–13] *views of mathematics* [14–16] or *epistemological beliefs.*[17–21] Each way of approaching to the views matches to specific theoretical and methodological traditions, but all of them agree that they are crucial elements in the interpretations of the events and that they influence the behaviour of individuals.

The general finding of these research domains indicates that people experience and give meaning to mathematics in many different ways. Particularly, the students' experiences as learners of mathematics at school and mathematics classroom are the main causes of their views of mathematics. Most of these studies are grounded on a quantitative approach based on survey methodologies (typically Likert scales). The questions of the surveys are constructed from conceptualizations about math, its teaching and learning that correspond

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to previous studies (e.g. [18,22–24]). However, these procedures impose the researcher's views on participants because they are forced to choose between predefined statements of mathematical views.

In contrast, there are other investigations that have been raised to understand the students' views of mathematics without imposing the researchers' meanings and concepts.[1–9,25] These studies were conducted using a *phenomenographic approach* to focus on how the people's experiences ascribe meaning to a specific phenomenon.[26] This approach allows the researchers to recover the voices of the participants and understand the importance of their social experiences in connection with their views of mathematics. All the international efforts made in this sense are reported in Europe and Asia, but in America the results are limited to Canada and the USA. We found no reports from Latin America. To fill this gap we conducted this research in order to ascertain the views that Mexican students have about mathematics, its teaching and learning. To investigate these perspectives, we made the following theoretical and methodological choices: (1) we chose to do a qualitative research study with the goal of not imposing our views to students and (2) we chose to use the concept of social representation (rather than belief or conception) considering that the points of view of the students are primarily the result of a social construction. In other words, we used the theoretical notion of *social representations* [27,28] because it is a form of *common sense knowledge* and the students' views are the product of a *social construction.*[29]

Thus, the research question of the present study is: *What are the social representations that Mexican high school students have about mathematics, its teaching and learning?*This question leads to the following questions: (1) What are their social representations about mathematics? (2) What are their social representations about teaching mathematics? (3) What are their social representations about learning mathematics?

2. Theoretical background

For the purpose of this research, we assumed that reality is a social construction that is transformed by the individuals. The social construction of reality [29] refers to the phenomenological tendency of people to consider subjective processes as objective realities. People apprehend everyday life as an ordered reality; they perceive reality as independent of their own apprehension, and imposed as something objectified.

> ...'reality' as a quality appertaining to phenomena that we recognize as having a being independent of our own volition (we cannot 'wish them away'), and to define 'knowledge' as the certainty that phenomena are real and that they possess specific characteristics. [29,p.13]

The *common sense knowledge* is the most basic knowledge of any individual as a member of a community, group or society. It is produced by the individuals in the social process of everyday life while interacting with others. At the same time, the community depends on the existence of this knowledge. In order to implement the above theoretical considerations, we use the construct of *social representations*,[27,28] a framework that allows the identification of the students' perspectives. Hence, we conceptualize the *social representations* about the social objects: 'mathematics', 'learning mathematics' and 'teaching mathematics', as forms of common sense knowledge.

Moscovici [28] developed the concept of social representations to understand the common, everyday knowledge of human beings about a specific social object. These are representations of reality formed and communicated among different groups of people, such as citizens in a community, colleagues in a working place, students in a university class or children and staff at a school. Each group produce, share and develop common ideas, norms, values and 'truths' because they share daily life. Social representations are practical knowledge as they are a guide for actions and social relations.[27] They are also a pre-decoding system of reality that establishes a set of anticipations and expectations.[30] Social representations include the set of beliefs, knowledge and opinions produced and shared by the individuals.[31] They are 'cognitive systems in which it is possible to recognize the presence of stereotypes, opinions, beliefs, values and norms that usually have a positive or negative behavioral orientation'.[32]

In summary, the concept of social representations has two socio-cognitive principles or hypotheses: (1) *the construction of reality* – every individual has a personal vision, a conception of the reality to which he or she belongs and (2) *the extent of social influence* – the people around the individuals influence their emotions, thoughts and behaviour. Moreover, social representations have four main characteristics: a social representation is (1) an organized set – the structure and the various elements that constitute the social relations are linked to each other; (2) shared by the individuals in the same social group – the partial consensus around it depends on the homogeneity of the group and on the position of individuals within the group; (3) collectively produced by means of a mass communication process; and (4) socially useful as a means of apprehending the object to which it relates. A social representation is a system for interpreting and understanding the social environment. Social representations also come into play in interactions between groups, particularly when those interactions are centred in social objects.

Some researchers in the field of mathematical education have used the concept of social representations to investigate social interactions in a multicultural/multiethnic mathematics classroom.[33–36] Broadly speaking, these investigations concluded that the individuals interacting in a classroom are all re-interpreting the different episodes from the perspective of the hegemonic social representations. We agree with Gorgorió and Abreu [33] that the notion of social representations may be easily applied to mathematical practices or mathematical teaching and learning since people interpret what happens around them as mathematical when it fits their image of what mathematics is. For example, teachers categorize their students as good, bad or indifferent based on their images of what is entailed in learning mathematics.

We also used the notion of metaphors to recover the voice of the students in this research. For Lakoff and Núñez,[37] metaphors are a way of conceiving one thing in terms of another and give expression to abstract realities in more concrete terms. The metaphor involves understanding one domain of experience in terms of a different one. The metaphor as a linguistic fact implies that a word or expression has a figurative sense used to call a reality with a name that does not correspond but helps to understand the domain in terms of another.

Several researchers have investigated the use of metaphors by teachers and students to know their beliefs about mathematics, its teaching and learning.[38–40] These researchers have found different metaphors about mathematics: mathematics as a *journey*, a *skill*, a *language* and a *puzzle*. Mathematics as a *journey* is one of the most common metaphors about mathematics that has emerged from research.[38–40] Mathematics can be 'a challenging journey and you get rewarded by arrival at your destination' or 'learning mathematics is like an easy stroll' or 'running uphill'.

Reader et al. [41] use three categories and their combinations: *production*, *journey* and *growth*. The most common metaphors were categorized as production. These metaphors indicated that students receive knowledge from their teachers in a passive way: 'teacher is as a sponge full of knowledge, squeezing it out into the empty glass'. Therefore, metaphors can allow students and teachers to understand and express abstract matters in concrete ways.[38] So the metaphors are linked to the processes of anchoring and objectification in the construction of social representations. According to Moscovici,[28] these processes are the ways in which new experiences, objects, bodies of knowledge or ideas integrate into a common thought by giving them meaning. The process of objectification translates 'abstract ideas and concepts into a concrete image or links them to concrete objects' [42,p.29] and 'objectify is reabsorbing one meanings excess materializing (and thus take some distance to your subject). Objectify is also transplanted to the plane of observation which was only interference or symbol'.[28,p.76] The process of anchoring provides that 'to anchor strange ideas, to reduce them to ordinary categories and images, to set them in a familiar context'.[42,p.29] Thus, the anchoring process allows insertion of the object of representation in a known and preexisting frame of reference.

3. Methodology

3.1. *Data gathering procedure*

Social representations are based on communications. Language contributes to maintaining and reinforcing the construction of social reality because:

> The language used in everyday life continuously provides me with the necessary objectifications and posits the order within which these make sense and within which everyday life has meaning for me [. . .] Language coordinates my life in society and fills that life with meaningful objects. [29,p.35,36].

The purpose of focus groups is to generate verbal narratives so it is an appropriate method to collect data in order to obtain social representations.

At the simplest level, a focus group is an informal discussion among a group of selected individuals on a specific topic.[43] We agree with the idea of Morgan that the advantage of focus groups as an interview technique lies in their ability to observe the interaction of the participants on a defined topic: 'Group discussions provide direct evidence about similarities and differences in the participants' opinions and experiences as opposed to reaching such conclusions from post hoc analyses of separate statements from each interviewee'.[44,p.10] This choice is also based on a previous research study at the same school where we observed that students felt more confident and comfortable to express their thoughts and feelings in this type of interview.

According to Morgan, a focus group is a 'research technique that collects data through group interaction on a topic determined by the researcher'.[45,p.130] This definition has three components. First, it clearly states that this method is devoted to data collection. Second, it locates the interaction in a discussion group as the source of the data. Third, it acknowledges the researcher's active role in creating the discussion group for data collection purposes. In a focus group, people are asked about their perceptions, opinions, beliefs, and attitudes towards a product, service, concept, advertisement, idea or packaging. Questions are asked in an interactive group setting where participants are free to talk with other members of the group. A focus group is intended to discuss and develop a topic or specific research event from the personal experiences of selected individuals.

We asked three questions in the focus groups for this research: (1) What does mathematics mean to you? (2) What does learning mathematics mean to you? (3) What does teaching mathematics mean to you? The role of the interviewer was to ask for more specific information and deepen in the use, meanings of words and phrases used by the students. That is why we asked secondary questions like: What do you mean by thinking logically? What do you mean by applying the knowledge? What do you mean by transmitting knowledge? What do you mean by understanding?

The focus group interviews were carried out during math classes in approximately one hour in a regular classroom and completely videotaped. During the calculus class, the teacher asked a group of three or four students to go to another classroom and participate as informants in a focus group interview in the research. We worked with six groups of three students and eight groups of four students.

3.2. *Participants and context*

The IPN (National Polytechnic Institute for its acronym in Spanish) is a public institution that provides free, or very low cost, education from the high school to postgraduate level in the area of science and technology. Its reputation as a quality option in free public education in the metropolitan city of Mexico causes that the IPN has many admission applications. The CECYTS (Centre for Science and Technology Studies for its acronym in Spanish) are part of the high school educational system offered by the IPN and are dedicated to the training of technicians. The IPN has 17 CECYTS distributed throughout Mexico City. The aspirants must apply a standardized test (Metropolitan Competition for Admission to high school) designed by the National Evaluation Center to evaluate their skills and knowledge and gain their admission to a public high school educational centre at the Metropolitan Zone. In practice, the score obtained in this test is used to allocate an applicant to a public school; thus, it is necessary to obtain a high score in the exam to enter to the most requested schools. The campus where the study was carried out required the lowest score compared to the other CECYTS.

Most students and participants in this study live in the bordering municipalities of the metropolitan area of Mexico City located in the state of Mexico. Most of them are from low economic extraction and their parents did not attend college level. Most of their mothers are housewives. Most of the students said that they want to pursue a career (engineering mostly). In optimal conditions, students take three years to complete the curriculum.

In order to achieve some homogeneity in the focus group, the participants were taken from a non-statistical sample of 50 students from the fifth semester of the chosen CECYT. They were from 16 to 18 years old and were attending the final part of the integral calculus course with the same math teacher. Due to the inflexibility of the curriculum, all students have the same schooling path composed of six courses with five hours each class per week: (1) algebra, (2) geometry and trigonometry, (3) analytical geometry, (4) differential calculus, (5) integral calculus and (6) probability and statistics.

3.3. *Data analysis*

The data analysis was performed in a constant comparative style,[46,47] which allowed the emergence of categories from the data. The central idea is that understanding emerges from an iterative process based on a constant sampling, comparison and analysis of transcribed excerpts from interviews. For data analysis, we used the ATLAS.ti software that allows qualitative analysis according to the constant comparative style.

The analysis involved two stages. In the first stage, the data analysis was made for each focus group separately, that is, in every interview transcript, we encoded each statement or dialogue about mathematics, its teaching and learning. During this process, we took special care of those dialogues and conversations that expressed agreement among participants. The second stage of the analysis was to group the statements, conversations and dialogues that had a similar meaning, even in different interviews, into a common category. After this, a set of categories was established and further intensive analysis of each particular category was carried out to identify a distinct meaning for them. These categories are not exclusives as they can have parts in common. Each category was used to identify the different social representations that were expressed by statements corresponding to the common sense knowledge associated with objects; this is why we have written these sentences beginning with the phrases 'Mathematics is. . .', 'To learn mathematics is. . .' or 'To teach mathematics is. . .'.

Videotaped interviews were fully transcribed. The students were identified with the labels *Sn* (with *n* being from 1 to 50). The *I* label identified the interviewer (the first author of this paper). We used a diagonal line between two words to note that two words have the same meaning from the perspective of students. Moreover, square brackets were used to establish a semantic equivalence between two phrases and between a phrase and a word. Thus, for example, daily/everyday indicates that for students the adjectives 'daily' and 'everyday' are equivalent, and apply/[put into practice] indicates a semantic equivalence between the word 'apply' and the phrase 'put into practice'. We identified these meanings' equivalence through secondary questions that the interviewer made to the participants.

4. Results

For the participants, *mathematics* has the function of solving daily life problems. Everyday life is an existential universe composed of at least three complementary sub-universes: (1) daily life at school, (2) daily life outside school and (3) the ideal life of employment associated with professions and technical specialties. In all these sub-universes mathematics is considered to be very important. In the world of school, the importance is highlighted by the view that mathematics is a subject that is the basis for other subjects (such as physics or chemistry). Outside of school, mathematics is considered necessary for a wide range of social practices related with numbers, size and business transactions. Furthermore, students said that to solve problems they had to use numbers and operations from the 'simplest' like addition, subtraction, multiplication and division, to more 'complex' as differentiation and integration.

Two characteristics are associated with mathematics: (1) mathematics is difficult and complicated, in the sense that it requires more dedication and time compared with other subjects and (2) mathematics is considered accurate, because the solution to an operation or a problem is unique.

For the students in this research, *learning mathematics* is closely linked to the vision of the role given to mathematics to solve problems of daily life. Students appeal to various metaphors in order to make sense of the phrase 'to learn mathematics'. In linguistic terms, one can observe that students use transitive verbs (have/acquire) where the action objects correspond to the functions conferred to mathematics. Thus, *to learn math* is (1) to possess/acquire/have knowledge to apply/[solve problems], (2) to [be able to]/[knowing how to] solve everyday problems, (3) to [be able to]/[know how to] make calculations and operations and (4) to reason/[think logically]/[have the ability] to solve problems.

In complement, *teaching mathematics* is closely linked to the metaphor of the transfer of possession from the person who teaches through the explanation. In linguistic terms, it can be observed that the verb 'to teach' is replaced with other transitive verbs where the action

Table 1. Students' social representations of 'mathematics', 'learning mathematics' and 'teaching mathematics'.

Mathematics is. . .
a way to [solve problems/[arrive at a result] of daily life (18)
calculations and operations with numbers (10)
important for daily life (11)
important for careers and for life (6)
complex and difficult (6)
important because it is in everything that surrounds us (4)
a subject that develops thinking skills/abilities (3)
exact (3)
To learn mathematics is. . .
to possess/acquire/have knowledge to apply/[solve problems] (10)
to [be able to]/[knowing how to] to solve everyday problems (10)
to [be able to]/[know how to] make calculations and operations (8)
to reason/[think logically]/[have the ability] to be able to solve problems (8)
To teach mathematics is. . .
to transmit/give/provide knowledge (17)
to know/understand to pass/share/give it (13)
to transmit/give the [reasoning ability]/[the understanding]/logic (11)
to [help]/[show as] solve problems (4)

object is the knowledge, [reasoning ability]/understanding/logic or [solve problems]. Thus, *to teach mathematics* is (1) to transmit/give/provide knowledge, (2) to know/understand to pass/share/give it, (3) to transmit/give the [reasoning ability]/[the understanding]/logic and (4) to [help]/[show as] solve problems.

In Table 1 are listed the social representations from the analysis of the narrative of the students. We have written in brackets the number of students in which we identify the respective social representation. It is possible to find none, one or more than one social representations in the narrative of one student (the sum of the numbers in square brackets is not a total of 50).

The aim of the following sections is to detail the social representations listed in Table 1. We also show evidence from the excerpts and underlined some expressions in order to emphasize them.

4.1. *Mathematics is important for daily life*

When asked about their ideas of daily life, the students referred to their day-to-day experience, such as buying and selling products, money management in transportation, personal expenses and measuring quantities.

> I: You also said that they help us with things in daily life, what sort of things?
>
> S49: For example, when cooking you need portions, and for the time <u>we need numbers</u>.
>
> S50: We can measure the time needed to cook our food, <u>things in life</u>.
>
> S49: When we buy something.

Others make reference to math as an important school subject that is useful in life and that mathematics is a tool for life.

S20: It is a very useful subject because we use them for everything else and because it is important for life and other subjects.

S23: A basic subject you use even in your daily life.

S21: It is a basic and indispensable subject in the life of any human being.

4.2. *Mathematics is important for career and for life*

When deepening on the usefulness of mathematics at school, students needed to clarify and made a distinction between what can be understood as mathematical tools for daily life and those that are useful in science and professional or technical careers or in workplace.

I: So, you think it is composed of various branches, can you give some examples?

S39: Trigonometry, Algebra, Calculus and Arithmetic.

I: Some of them are used daily, for example, those you have mentioned, where do we use them?

S39: Well, we use Arithmetic everywhere, at supermarkets. . . and the others

I: and Trigonometry?

S38: It also depends, for example certain careers, in your job, you can use them.

The previous testimonies about the importance of math in daily life are complemented with references to the ideal world of technical and professional careers and to the assumption that mathematical knowledge is fundamental or important.

S26: It is one of the basic sciences for any professional career, because we use them in our daily life.

S25: It is not only important for a career, but also in the daily life.

S27: First, it is basic for all your studies, no matter what your career will be, and also because they are part of your learning process.

We found arguments about the utility of some specific mathematical context when the students mentioned the usefulness of mathematics in their future careers. The following dialogue shows their relation to the concept of infinitely small numbers:

S3: As S2 was saying, everything is involved with mathematics; we need to use them everywhere, so we need to keep updating to go on.

I: Will you apply outside everything you are learning now?

Everyone: Yes.

S1: For everything, for example, how is it possible to know the distance between Earth and Mars? There are no rules to measure from here to there.

S2: Yes, that is true.

S1: For example in the case of textile engineering, the way they know how many threads and strings are in a complete sweater from just a 3 cm. sample. Then they are just hypothetical things because you don't know, but you can reach scientifically if you use infinitely large or small numbers.

4.3. *Mathematics is important because it is in everything that surrounds us*

The idea that mathematics is in 'everything that surrounds us' prevails in a superlative degree. The following dialogue gives an idea that mathematics is in design, construction and nature:

I: Then you said that it allows us to find the function of everything that surrounds us, what sort of things are you referring to?

S2: Like cars, tables, from the moment a table is designed, you measure his volume; I don't know, everything that surrounds has to be with math.

———

I: By the way, you said that you learn to solve the mysteries of the universe, how do you think that those mysteries could be solved?

S31: For example, in Physics we use math formulas... we use math expressions and we explain every physical or chemical change with them, in all subjects.

4.4. *Mathematics is a way to [solve problems/[arrive at a result] of daily life*

For students the central idea is that the function of mathematics is to solve problems of daily life or problems in general. The view that mathematics is used to solve problems of daily life can be interpreted as a complement of the image of mathematics as important and essential to everyday life. When questioned about the daily life problems, we found that the students referred to their daily personal experience, which includes the use of some sort of 'simple' mathematics.

S41: Also with family, as a family member, it could be good to be clear at or to know math to administrate and take care of my family or myself.

I: In particular, do you really use math? Where do you use it?

S42: It could be, for example, in what he said, in administration, how much will you spend? How much?

S41: Taking percentages, I mean we use them more in economical issues, almost everyone does it, to take percentages or time limits at shopping, also in math, the simple ones.

For some of the students, the meaning of 'daily life' includes the ideal world of profession, work and technical specialities. Again, it could be noted the representation of the ideal professional that they wanted to become and the bonds between mathematical knowledge and employment, particularly in engineering, as is evident from the following dialogue:

I: 'Daily problems', of what kind?

S48: Since we go to the supermarket, at the employee. . .

I: Employee?

S48: Engineers.

S46: You can use them in some works, from a simple addition, trucks.

S48: The one who sells fruits on the street.

S47: Anything, from a figure.

4.5. *Mathematics is calculations and operations with numbers*

In a generalized way, the students evoke the actions of calculating and making 'basic' and 'complex' calculations. They also evoke the different 'branches of mathematics'. In this way, most of the calculations can be of numerical type from addition and multiplication to advanced operations such as deriving and integrating functions.

SI: To what type of calculations do you refer?

S36: All mathematical operations.

I: Any addition, subtraction, multiplication, division, integration, derivative.

S35: Root, power. . .

4.6. *Mathematics is complex and difficult*

The characteristics granted to math make reference to some qualities such as 'complex' and 'difficult'. The 'complex' aspect of math is associated with the level of care required and the time necessary to do what is done in math. When the students were asked about the reasons why they considered that math requires time, some students said that math requires more attention and dedication than other subjects, and that all their classmates share this idea.

S10: Math is a complex subject that needs time.

S11: And it is also complicated and hated by the most, although I do not hate.

S12: It requires attention, time and patience for reasoning, thinking and memorizing.

I: Do other subjects require attention?

S10: Yes.

Everyone: Some.

S11: But we are talking about math, that's why we said that.

S12: Yes, it is a fact that this requires most of our attention; we all get nervous and spend time on it.

S11: We all have difficulties with numbers.

4.7. *Mathematics is exact*

When the students were asked to clarify their meaning to exactitude, a couple of students affirmed that it refers to the fact that math results cannot vary by context.

I: What do you mean by exact?

S32: That two plus two is always four, here and in China.

S31: It can never vary.

Other students matched the meaning of precision with the idea that a mathematical problem has only one answer arguing that the algorithms produce a unique solution.

S10: Because when you solve a problem there is just one answer, generally, for example, there is only one answer in an equation, there are not many.

4.8. *Mathematics is a subject that develops thinking skills/abilities*

This representation is structured around the meaning that students granted about the things that mathematics develops. Some students consider that mathematics develops thinking skills/abilities that allow for the development of reasoning/[logical thinking].

Some students refer to ability as the 'mental skills' associated with reasoning or logical discernment needed to solve a problem.

S1: For example, the approach to any problem, the discernment, I mean you make the approach to the problem, you understand it, then you search for alternatives to solve it. . . that is a mental skill to any person because it is not only mathematics.

Other students express that a skill/ability is to be able to solve daily life problems in a simple 'way', without any difficulty (remember that being difficult is a characteristic associated with the time needed to solve something) and that reasoning is a skill/ability that allows to solve problems and to perform the math operations associated with them. The following dialogue shows the construction of the chain of designated meanings:

I: How do you believe those abilities are developed?

S26: Reasoning... what's the name?

S24: The ability to do something.

S26: To solve a problem.

S25:In a simple manner, it is a skill because...

S26: Apply it in daily life can be a skill.

4.9. *To learn mathematics is to possess/acquire/have knowledge to apply/[solve problems]*

This social representation is linked to metaphors of learning that are related to possessing/acquiring knowledge that can be applied in problem solving. In this way, learning is conceptualized as the acquisition of a possession. This view of learning would then be a pragmatic characterization of learning which considers that learning is to acquire or voluntarily acquire knowledge that someday could be evoked to solve everyday problems. Under this idea if knowledge does not help to solve problems, then it would not have any function or utility, which is demonstrated by the following dialog:

S15: Learn math is to have knowledge about them and be able to apply it.

S18: Learn math is to acquire knowledge, to learn how to deal with situations that we put on the road, because human beings depend on mathematics.

S17: To know what we listen and be able to put it into practice.

The link between applying and solving everyday problems or tests was perceived when they were asked to be more precise about the meaning of 'applying mathematics'.

S44: We learn and then apply what we learn, voluntarily or involuntarily.

S43: In a test.

S44: In everyday life.

4.10. *To learn mathematics is to [be able to]/[knowing how to] to solve everyday problems*

Similarly to the previous representation, this one is also linked to the ability to solve problems. The students' representation of 'problems' is associated with the routine or daily life problems and consists of three facets: the daily school, the everyday after school and the everyday of the ideal image of being professional or an employee. It can be noticed that for the students the notions of exercise and problem are equivalent.

> S25: Learn mathematics is to learn different ways of solving problems, equations and that sort of things.
>
> I: How do you learn math?
>
> S24 and S26: Practicing.
>
> S26: Well, yes, practicing the exercises that they teach us and the ones that they teach us to apply in the daily life. That's how I could learn; I think that's the way we could learn better.

The use of metaphor *to solve problems* as an objectification for learning math establishes that the process of resolution of problems and the process of learning are equivalent for the students. In the next dialogue, a nuance of the previous statement can be observed; the student said that someone who is solving a problem is in the process of learning math.

> S37: If you solve a problem, you are in the process of learning math.
>
> I: When I reach to the result I have finished the process?
>
> S38: No, it is just like that, that's why they [*the teachers*] put us several exercises, they exemplify.
>
> S37: It is practice.

4.11. *To learn mathematics is to [be able to]/[know how to] make calculations and operations*

This social representation is related to some of the most visible objects of mathematics: the calculations and numerical calculus. From this point of view, to make something mathematically corresponds to making calculations. The students of the current study mentioned that there are several kinds of operation, like the derivative calculus. This shows that the operative nature of mathematics and its association with its learning is a product of a long process lived at school. The following testimonies show that the students evoke actions of calculating and making 'basic and complex' operations of different 'branches of mathematics'.

> S6: Learn mathematics it is to learn to perform calculations with the mind more and more advanced.
>
> ——
>
> S20: The knowledge of where and how to apply the mathematical calculations besides of the knowledge of how to solve the calculations.

4.12. *To learn mathematics is to reason/[think logically]/[have the ability] to be able to solve problems*

This social representation is linked with the learning metaphors related to a capacity/ability of thinking/reasoning to be able to solve problems.

From this point of view, the possession that has to be acquired is the reasoning/[the logical thinking].

I: Is it to learn and develop logic?

S32: Developing logic is like all you are learning. Obviously mathematics in all its concepts is a logical thing, everything has a logic, everything has a reason, and development, i.e. while you are learning math you develop logic little by little, or not? To understand quite well how it all works and how it is ruled.

We found a link between this social representation and the ability of solving problems when we asked the students to clarify the meanings of these capacities/abilities. This is why the metaphor of understanding relies on problem resolution. So, learning is 'reason/[think with logic]/[have the ability] to solve problems'. The following dialogues are samples of this social representation:

S43: I solve a problem logically.

S44: From the very basic, mathematics is applied in daily life.

S43: Using more the logic.

—

I: What is developing logic?

S1: To develop logic is like everything you are learning.

4.13. *To teach mathematics is to transmit/give/provide knowledge*

This social representation is structured with the use of metaphor to transmit/give knowledge as a reference for teaching mathematics. On grammatical terms the verbs that substitute 'to teach' take knowledge as the action object. In this way the knowledge is conceptualized like a good or possession of a person that can be transferred to another through the will of the first one, which is demonstrated by the following dialogues:

S1: Teach is to transmit the theoretically acquired knowledge and personal experiences to the students in an interesting and nice way, so that they will become easier to learn.

S2: To give another one a little or a lot of your scientific knowledge, but also the knowledge acquired through time.

—

S6: To transmit the little knowledge I have from the topic to help others.

S8: To share the previous acquired knowledge with the dominated one and we could show them to other people.

The way in which the young students conceptualized the acquisition of knowledge transference was through the explanation of the one who knows/[has the knowledge] in a detailed/[step by step] form. According to the students, that explanation should be adapted to the knowledge of the receiver, meaning that, following the metaphor of the transference of a possession, the one to whom the explanation goes should be prepared to receive the knowledge, and the one who explains must verify the reception, as shown by the following dialogue:

S3: Well, I believe that from the point of view of a teacher, before you go and stand in front of a public you should study the public you will talk to.

S1: A diagnosis.

S3: It is not the same to talk to high school students than to talk to primary school students.

4.14. *To teach mathematics is to know/understand to pass/share/give it*

This social representation is complemented in a direct way from the previous one. Here, the students make explicit that the one who teaches must possess knowledge. In this way, the metaphor of teaching mathematics as a transfer of a personal possession is extended; moreover, it is used as an interpretative frame to classify the performance of the one who teaches (mainly a professor) in three categories: (1) one who does not know/domain, (2) one who does know/domain and knows how to explain and (3) one who only knows/domains math, as shown in the following dialogue:

S14: To know well what you do and understand it perfectly so you can share or express with other people.

S15: To teach means to be a good mathematician so the others could learn about my acquired knowledge.

S16: For teaching math it is needed someone who teaches us, someone that dominates math and knows how to explain it; because the knowledge that he dominates is useless if he lacks of the knowledge to explain them, how to solve them or how to apply them.

4.15. *To teach mathematics is to transmit/give the [reasoning ability]/[the understanding]/logic*

In this social representation, the metaphor of math teaching is taken as a transfer and the possession is the ability [reasoning ability]/[the understanding]/logic. In this way we found again the metaphor about teaching as the transfer of an individual's possession to another one who does not have it through the will of the first one. The action object corresponds to the reasoning ability or the understanding. The following testimonies show the above:

S39: To transmit the understanding to other.

S12: Someone that besides understand them completely, knows how to make others understand them the same way.

S11: Transmit and make other people understand.

S17: To transmit and make other people use their reasoning abilities in a problem.

4.16. *To teach mathematics is to [help]/[show as] solve problems*

In this social representation, the metaphor of math teaching as a transfer is maintained and the possession is the knowledge of the procedure/solution of a problem. This view corresponds to the idea that knowledge is attained by imitation, i.e., through the reproduction of what is seen or heard, in this case solving problems, and showing that the proof of knowledge is that the learner had not demonstrated that behaviour before. The following testimonies show this:

S13: For me, teaching is to show the knowledge or the different steps and ways of solving things (exercises or that sort of things) to the others.

S22: To show to the students to solve any problem they have.

S25: To share knowledge with other people and clarify the steps or details of any exercise where doubt remains.

S28: To show how you can solve problems.

5. Conclusions and discussion

The main focus of this research was to collect data through focus group interviews. The unique strength of this technique is the possibility to observe the scope and nature of agreement and disagreement among the interviewees.[44,45] This allows us to pay more attention to the meanings of the words and phrases used by the students to categorize them in the different social representations. The relative consensus that we found, not only among the students inside a focus group, but also from different interviewees, shows evidence of the existence of widely shared social representations, even if we lack statistical significance.

The student participants are in the process of objectifying mathematics. This can be shown contrasting their social representations of mathematics with the ones for its teaching and learning. It is remarkable that students conceptualize mathematics using the 'way of thinking' metaphor while talking about the teaching and learning of mathematics but not while responding for the meaning of math. For the latter, they focused on describing and highlighting the importance and utility of mathematics but hardly saying anything about it. The broader conceptions for mathematics arose from their metaphors for learning and teaching because they are conceptualizations of learning/teaching 'what mathematics is', e.g. the social representation 'mathematics is a way of thinking or reasoning' can be derived from 'learning mathematics is learning to think logically'.

As in other research studies,[4–8] we find conceptions of mathematics ranging from *narrow conceptions* (mathematics are calculations with numbers) to *broader conceptions*(mathematics as a way of thinking) or from *fragmentary conceptions* (mathematics is numbers, rules and formulae... which can be applied to solve problems) to *cohesive conceptions* (mathematics is a complex logical system and way of thinking... which can be used to solve complex problems... and provides insights used for understanding the world).[1–3]

The main social representation of mathematics among the participants is that it is a way to solve everyday problems. Practically all the referenced studies reported that, for students from middle to high school, mathematics is numbers and operations to solve problems. In contrast, we did not find the social representation of mathematics as a system of knowledge [8] or as models.[9] This can be explained by taking into account that the participants have not yet worked with explicit mathematical models in this educational level.

The similarities and differences with the results found by [4–8] can be explained by the fact that the participants' experience with mathematics is more limited than that of college students participating in such researches. Wood et al. [8] found that university students are more likely to describe broader conceptions of mathematics in the last years of study, mainly between the second and third year. In the same way, the international study of math conceptions of undergraduate mathematics students of Petockz et al.,[4] which polled 1200 students from five countries (Australia, South Africa, Canada, Brunei and Northern Ireland), showed that broader conceptions of mathematics were more likely to be found in later years.

Most narratives about mathematics are a kind of apologia of mathematics as a priority field of knowledge and its usefulness in different contexts and situations. This is consistent with the main role assigned to mathematics in the IPN curriculum from the high school to graduate level. Particularly, it is the subject with a maximum number of hours/classes assigned in high school. On the campus where the field work was conducted, aimed at physics and mathematics, students devoted five hours per week during each of the three years. This constant presence necessarily reinforced their appreciation of mathematics as an important field of knowledge.

Other researchers have also found these conceptions in students in different academic levels. Houston et al. [9] found that among American secondary students mathematics is important, difficult and based on rules. Kislenko [12] found that among 245 Estonian students of upper secondary school (ages 16–19, who had chosen the advanced mathematics course or the one which focuses on more practical applications of mathematics) 91% agreed that mathematics is important and 81% agreed that mathematics is useful in their lives, but tends to be difficult and boring. Kaldo [16] found that Estonian science university students think that knowledge of mathematics is important to understand the world and that they study mathematics because they know how useful it is. In the sample of this research, 81% of the students mentioned the importance of mathematics in everyday life. Our research shows similar results to Kaldo [16] in the sense that students have positive values towards mathematics, especially if they are students who choose to study mathematics beyond compulsory education.

It has also been reported in other investigations that students from different academic levels believe that mathematics is difficult. Kislenko [12] described that approximately 65% of the upper secondary school participants expressed that mathematics is difficult against 35% who thought it is easy in general. Zan and Di Martino [48] asked Italian students of all academic levels to write an essay entitled 'Me and mathematics: my relationship with maths up to now'. They identified in their analysis that many of the students characterized mathematics as useful/useless and easy/difficult. These dichotomies are frequently used in questionnaires about attitude towards mathematics.

We also established some similarities and differences with the results of a research conducted by Crawford et al. [1] with students in their first year of college. We found that the social representations of learning mathematics are consistent with the *superficial approach to learning mathematics* (attention and activity focus on the reproduction of knowledge transmitted by experts). In contrast, we did not find any social representation of learning mathematics corresponding to the *deep approach to learning mathematics* (personal and global perspectives on learning strategies aimed to give sense to new information from pre-existing conceptions or opinions). This happens because the methodology does not allow the exploration of the students' learning approaches, i.e. how learning is accomplished.[1–8] We asked for the meaning of the abstract concept of 'learning mathematics' but not how to achieve it.

The learning conceptions found here are also consistent with other research works that have delved into the teaching metaphors used by teachers of mathematics. For example, 'most of the experienced and prospective teachers in Martínez et al. [49] shared the traditional metaphors depicting teaching and learning as transmission of knowledge. Future research may explore these metaphors of students and teachers in more detail, as suggested by Reeder et al. [41]

Moreover, we found some differences in the metaphorical thinking of the students in this research. The metaphor 'learning math is learning to think logically... to solve problems' does not assume knowledge as an object but as a process of thought: 'think logically'. The presence of this metaphor among students indicates that the representation 'math is a way of thinking' is under construction. Future research may delve in the role of metaphorical thinking of students in shaping their social representations of the specific social objects considered here. These studies can be performed with reference to the production processes of representations: objectifying and anchoring.

It is not surprising that most of the results in this research are consistent with other phenomenographic investigations.[1–9,25] This is clearly explained by the connections between the theoretical and methodological approaches. Richardson [50] highlighted the broad similarity between the grounded theory and the phenomenographic approach: both types of research focus on the narratives of students without imposing predefined views by the researchers and analyse data in a similar way. All results as a whole clearly indicate some of the components for a *phenomenology of mathematics.* This is understood as the ways mathematics is perceived and interpreted as a phenomenon by students.

In phenomenological terms, the presence of similar students' conceptions in different countries and cultures leads to the possibility that students around the world have similar mathematical experiences in school and mathematical classrooms. However, this research is based on the analysis of the narratives. This type of data analysis cannot access the details of the students' conceptions, so we could not identify 'Mexican conceptions' of mathematics. For example, we can find the idea that 'learning mathematics is learning to solve problems' in different countries. This idea does not detail the type of problems that the students are concerned with, or how the mathematical knowledge comes into play to solve problems. From this point of view, the similarities found with the conceptions may be apparent. As we have seen before, a complementary hypothesis is that the similarities are due to the fact that the investigations were carried out under a phenomenographic approach. Future research could work out these hypotheses.

We believe that the results of this research could be generalized to a larger group of students, such as Mexican high school students or CECYT-IPN students even if the sample was too small. The *theoretical saturation* concept could be useful for achieving generalizable results. Glaser and Strauss stated that 'Saturation means that no additional data are being found whereby the sociologist can develop properties of the category'.[47,p.61] We find social representations from consensus not only among students inside one focus group but between different focus groups. In this regard, it will be interesting to investigate more about the students' views of mathematics with other homogeneous groups of students to understand the differences and similarities.

From the previous discussions we can argue that the concept of social representations is useful to identify the social constructions of the students' views of mathematics, its teaching and learning. The consensus on the opinions of the students sustains this statement. This consensus is based on the socio-cognitive hypotheses underlying the concept of social representations (construction of reality and extent of social influence). This shows that CECYT is an institution (in the sociological terms of Berger and Luckmann [29]) with a mathematical culture reproduced by teachers and internalized by students throughout their school trajectory. This mathematical culture takes the form of the social representations found in this research. Future research can use the construct of social representations to delve into this sociological and cognitive approach to the source of the conceptions of students and teachers.

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